

Assignment 10

Coverage: 16.5 (skip “Implicit Surfaces”) and 16.6.

Exercises: 16.5 no 4, 8, 10, 13, 20, 24, 31, 33, 38, 42, 45; 16.6 no 4, 8, 10, 13.

Hand in 16.5 no 8,33; 16.6 no 8, 13 by Nov 22

Supplementary Problems

1. Consider the parametric surface

$$\mathbf{r}(u, v) = (u + 6v, -2u - 12v + 5, -1), \quad (u, v) \in [0, 1] \times [0, 1] .$$

Is it a smooth surface? Describe its image. Recall that by definition a parametric surface is smooth if \mathbf{r} is continuously differentiable and $\mathbf{r}_u \times \mathbf{r}_v$ is linearly independent in the interior of the region of definition.

2. Let S be the surface of revolution obtained by rotating $\mathbf{r}(t) = (f(z), z)$, $f(z) > 0$, $z \in [a, b]$ around the z -axis. Show that its surface area is given by

$$2\pi \int_a^b f(z) \sqrt{1 + f'^2(z)} dz .$$

Derive this formula using Riemann sum approach. Hint: Consider the cross sections along the z -axis.